Frequency Shift in the Equatorial Plane Rotating Ayon-Beato-Garcia of Black Hole Spacetime

Ravi Shankar Kuniyal
Department of Physics, Government PG College, Gopeshwar (Chamoli), India
E-mail: raviskuniyal@gmail.com

INTRODUCTION

Black holes (BHs) are one of the most fascinating objects in the universe, obtained as exact solutions of Einstein's field equations in general relativty (GR) [1, 2]. The most general spherically symmetric, vacuum solution of the Einstein field equations in GR is the Schwarzschild BH (SBH). A static solution to the Einstein-Maxwell field equations, which corresponds to the gravitational field of a charged, non-rotating, spherically symmetric body is the Reissner-Nordstr¨m spacetime. Further, the rotating generalization of the SBH spacetime is Kerr BH spacetime, while theo spacetime geometry in the region surrounding by a charged rotating BH is represented by the Kerr-Newman BH [1, 2]. The rotating BH solutions in GR are interesting from the astrophysical point of view and work as the key to understand the most vibrant phenomena in the observed universe. The rotating Ay´n-Beato-Garc´ BH is an interesting BHoıa solution of Einstein equations coupled to nonlinear electrodynamics. Rotating Ay´n-Beato-Garc´ BH is a specialoıa type of regular BH which is continuous throughout spacetime [3, 4].

ROTATING AYON-BEATO-GARCIA BLACKHOLE SPACETIME

ThelineelementofrotatingAyo´n-Beato-Garc´ıa(ABG)BH,is described by the following metric in the Boyer-Lindquist coordinates [4],

$$\begin{aligned} ds^2 = & -f(r,\theta)dt^2 + \frac{\Sigma}{\Delta}dr^2 \\ & - 2\,a\,sin^2\theta\,\big(1 - f(r,\theta)\big)d\phi\;dt + \Sigma d\theta^2 \\ & + sin^2\theta\,[\Sigma - a^2(f(r,\theta) - 2)sin^2\theta\,]d\phi^2, \end{aligned} \qquad \text{...(1)}$$

with,

$$f(r,\theta) = 1 - \frac{2Mr\sqrt{\Sigma}}{(\Sigma+Q^2)^{\frac{3}{2}}} + \frac{Q^2}{(\Sigma+Q^2)^2},$$

$$\Delta = \Sigma f(r,\theta) + a^2 sin^2\theta\,,$$

$$\Sigma = r^2 + a^2cos^2\theta. \qquad \text{....(2)}$$

In above Eqn. (2), a, M and Q are rotation, mass and charge parameters respectively. For $Q = 0$ the metric given by Eqn. (1) reduces to a KBH and in addition to this, if $a = 0$ then the metric becomes a SBH in GR. The horizons (inner and outer horizons) of rotating ABG BH are calculated as [For more details refer [3, 4]],

$$\Sigma f(r,\theta) + a^2 sin^2\theta = 0. \qquad \text{...(3)}$$

The above Eqn.(3) shows the angular dependency of horizons.

2.1 Effective potential

The study of effective potentialisa very useful tool for describing the motion of masslesstest particles and the various types of orbits associated with them. In the equatorial plane (i.e. $\theta = \pi/2$), the metric (1) can be written as,

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + C(r)\,d\phi^2 - D(r)\,dt\,d\phi, \qquad \text{...(4)}$$

where the metric coefficients are described as below,

$$A(r) = 1 - \frac{2\,M\,r^2}{(r^2+Q^2)^{\frac{3}{2}}} + \frac{Q^2}{(r^2+Q^2)^2}, \qquad \text{...(5)}$$

$$B(r) = \frac{r^2}{r^2\,A(r)+a^2}, \qquad \text{...(6)}$$

$$C(r) = r^2 - a^2[A(r) - 2], \qquad \text{...(7)}$$

$$D(r) = 2a[1 - A(r)]\,. \qquad \text{...(8)}$$

The first integral of null geodesic equations may be expressed in terms of the above mentioned metric coefficients [5, 6], in the following form,

$$\dot{t} = \frac{4\,C(r)E - 2\,D(r)L}{4\,A(r)C(r) + D^2(r)}, \qquad \text{...(9)}$$

$$\dot{r} = \pm\sqrt{\frac{C(r)E^2 - D(r)E\,L - A(r)L^2}{B(r)(4\,A(r)C(r) + D^2(r))}}, \qquad \text{...(10)}$$

$$\dot{\phi} = \frac{2\,D(r)E + 4\,A(r)L}{4\,A(r)C(r) + D^2(r)}. \qquad \text{...(11)}$$

Here, E and L are energy and angular momentum of massless particles respectively. Fornullgeodesics, $\dot{r}$ fromEqn. (10), can be reconstructed as,

$$\dot{r}^2 + V_{eff} = 0\,,$$

where,

$$V_{eff} = -4\left[\frac{C(r)E^2 - D(r)E\,L - A(r)L^2}{B(r)(4\,A(r)C(r) + D^2(r))}\right],$$

$$\begin{aligned} = & \frac{1}{r^2(Q^2+r^2)^{5/2}} \times [2aLEr\,(-Q^2\sqrt{Q^2+r^2} + \\ & 2Mr\,(Q^2+r^2)) + L^2(-2Mr^2(Q^2+r^2) + \\ & \sqrt{Q^2+r^2}(Q^4 + r^4 + Q^2r(1+2r))) - \\ & E^2(r^2(Q^2+r^2)^{5/2} + a^2(2Mr^2(Q^2+r^2) + \\ & \sqrt{Q^2+r^2}(Q^4 + r^4 + Q^2r(-1+2r))))] \end{aligned} \qquad (12)$$

The effective potential depends on the two conserved quantities energy E and angular momentum L of the particles respectively. The general behavior of effective potential as a function of r for different values of rotation parameter a is presented in Fig. (1). In the potential plot (see Fig.(1)), the large dashes and solid line represent the case of SBH and KBH respectively. From the graph of effective potential one can observe the effect of rotation parameter on potential. On increasing the value of rotation parameter the plot of potential shifts to wards right which signifies the shifting of circular orbit away from the central object. It is also found that the orbit is more stable in case of rotating ABG than SBH and KBH due to the presence of rotating parameter a.

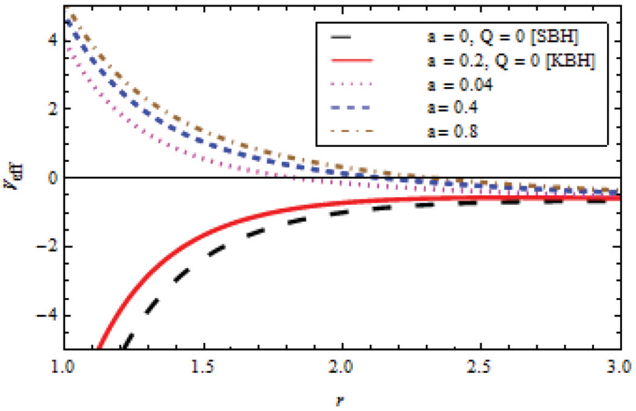

Figure-1: Variation of effective potential V_{eff} with radius at different values of rotation parameter a (for L=3, E=1, Q=1).

FREQUENCY SHIFT

Here, as one of the optical phenomena, the combined (gravitational and Doppler) frequency shift for rotating ABG BH is analyzed in the equatorial plane. In order to compute the red shifts that photon experiences, we follow [7, 8]. The frequency shift g is the ratio of observed photon energy E_0 E_0 to emitted photon energy E_e [7, 8], expressed as,

$$g = \frac{E_0}{E_e} = \frac{(k_0)_\mu u_0^\mu}{(k_e)_\mu u_e^\mu}. \qquad \text{…(13)}$$

In above Eqn. (13), $(k_0)_\mu$, $(k_e)_\mu$, are components of photon four-momentum at the event observation and u_0^μ, u_e^μ are component of the four-velocity of the observer. The four-velocity for static distant observer reads u_0 =(1,0,0,0). For the emitter following a circular geodesic at $r=r_e$ in equatorial plane of the given BH spacetime, the four-velocity reads $u_e = (u_e^t, 0, 0, u_e^\phi)$. The components u_e^t and u_e^ϕ are as,

$$u_e^t = \frac{1}{-2Mr^4(Q^2+r^2)+\sqrt{Q^2+r^2}\left(a^2(Q^2+r^2)^2+r^2(Q^4+r^4+Q^2r(1+2r))\right)} \times$$
$$\left[aEr\left(-Q^2\sqrt{Q^2+r^2}+2Mr(Q^2+r^2)\right)+ L\left(-2Mr^2(Q^2+r^2)+\sqrt{Q^2+r^2}\left(Q^4+r^4+Q^2r(1+2r)\right)\right)\right] \qquad \text{...(14)}$$

$$\Omega\, u_e^\phi = u_e^t \qquad \text{…(15)}$$

In Eqn. (15), $\Omega = \frac{d\phi}{dt}$ is the angular velocity of the circular geodesics relative to distant observers. The expression of angular velocity comes out as,

$$\Omega = \frac{4\,a\,L\,r\left[\frac{Q^2}{(Q^2+r^2)^2}-\frac{2Mr}{(Q^2+r^2)^{3/2}}\right]+4\,E\left[r^2+a^2\left(1-\frac{Q^2r}{(Q^2+r^2)^2}+\frac{2Mr^2}{(Q^2+r^2)^{3/2}}\right)\right]}{4\,a\,E\,r\left[\frac{-Q^2}{(Q^2+r^2)^2}+\frac{2Mr}{(Q^2+r^2)^{3/2}}\right]+L\left[4+\frac{4Q^2r}{(Q^2+r^2)^2}-\frac{8Mr^2}{(Q^2+r^2)^{3/2}}\right]} \qquad \text{...(16)}$$

Now, the frequency shift given by Eqn. (13)comes out in the following form,

$$g = \frac{1}{\left(1-\frac{L}{E}u_e^t\right)\Omega}. \qquad \text{...(17)}$$

Here, u_e^t and Ω are given by Eqn. (14) and Eqn. (16) respectively. The variation of frequency shift for different values of rotating parameter is depicted in Fig.(2), where the large dash and solid line represents the case of SBH and KBH respectively. One can see that on increasing the value of rotating parameter the frequency shift decreases. The decrement of frequency shift with rotation parameter shows the weakness of gravitational field around rotating BH.

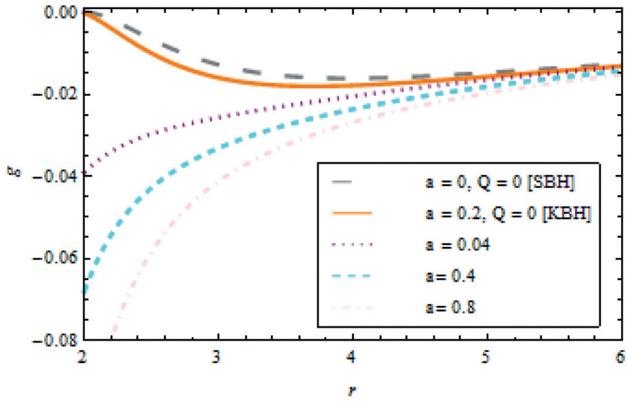

Figure-2: Variation of frequency shiftgwith radius at different values of rotation parametera (for L=1, E=1, Q=1, M=1)

SUMMARY AND CONCLUSIONS

We have discussed the nature of effective potential for massless particles in equatorial plane of rotating ABG BH spacetime and calculated the frequency shift of photons. Few of the important results obtained are summarized below.

- The nature of effective potential changes with the increasing values of rotation parameter. The circular orbit shifts away from the central objects and hence the attractive nature of the effective potential changes accordingly.
- The gravitational field of a rotating ABG BH is found to be less attractive in nature than SBH and KBH i.e. the rotation parameter weakens the attractive nature of gravitational field of rotating ABG BH.

The results obtained herewith can be useful in the study of gravitational lensing phenomenon around the rotating ABG BH spacetime in future.

REFERENCES

1. Hartle, J. B.: Gravity: *An Introduction to Einstein's General Relativity*. Pearson Education Inc., Singapore, 2003.
2. Chandrasekhar, S.: *The Mathematical Theory of Black Holes*. Oxford Uni. Press, New York, 1983.
3. Toshmatov, B., Ahmedov, B., Abdujabbarov, A. and Stuchlik, Z.: *Rotating Black Hole Solution*. Phys. Rev. D89, 104017 (2014).
4. Ghosh, S.G., Sheoran, P. and Amir, M.: *Rotating Ay´on-Beato-Garc´ıaBlack Hole as a Particle Accelerator*. Phys. Rev. D90, 103006 (2014).
5. Wei, Shao-Wen., Liu, Yu-Xiao., Fu, Chun-E and Yang, K.: *Strong Field Limit Analysis of Gravitational Lensing in Kerr- Taub-NUT spacetime*. JCAP. 1210, 053 (2012).
6. Fernando, S.: *Null Geodesics of Charged Black Holes in String Theory*. Phys. Rev. D85, 024033 (2012).
7. Stuchlik, Z., and Schee, J.: *Circular Geodesic of Bardeen and Ayon-Beato-Garcia Regular Black-Hole and No-Horizon Spacetimes*. Int. J. Mod. Phys. D24, 1550020 (2014).
8. Schee, J., and Stuchlik, Z.: *Profiles of Emission Lines Generated by Rings Orbiting Braneworld Kerr Black Holes*. Gen. Rel.Grav. 41, 1795-1818 (2009)